Chaos as a basis of new principle for detecting the gravitational waves

George Vlasov
Landau Institute for Theoretical Physics and
Moscow Aviation Institute,
Moscow, Russia*

February 5, 2008

Abstract

A particular example of chaos can be conceived in the interaction of non-linear oscillator with a harmonic gravitational wave. When we replace the linear potential forces by the therm SIN(x), the type of solution becomes subject to external perturbation. Although the perturbation produced by the gravitational wave is weak the standard estimations allow to predict the appearance of chaos at definite range of parameters. This qualitative change in the character of motion immediately detects the fact of impact with gravitational wave. Another advantage relates to a broad range of frequencies so that the narrow resonance band is not required.

The gravitational wave detectors [1, 2, 3] are under intent achievement for almost thirty years. Laser interferometry [4, 5, 6, 7, 8, 9] and resonant mass antennas [10, 11, 12, 13, 14, 15, 16] are considered as the most perspective approaches on the way of discovering gravitational radiation. As a matter of fact, resonance in a vibrating system, whichever different phenomena give rise to vibration, is the basic principle of this approach. Thus, a higher sensitivity of the detector, a higher amplification and a

*E-mail: vs@itp.ac.ru

frequency close to that of the gravitational wave, not known beforehand, are required. These are the most difficulties which inevitably impede the researchers. In the present study we propose a principally new method for detecting the gravitational waves. Instead of resonance in the linear oscillator we appeal to a non-linear system vibrating near the separatrix. The gravitational wave, then, may set a system to a dynamic chaos directly distinguished from the former oscillatory regime. With it, this qualitative response occurs even for a considerable (appreciated by orders) difference in frequencies of the oscillator and the external agent.

Let a linear polarized gravitational wave propagate along axis x and collide with detector. As a particular instance which admit a wider application we consider a pendulum oscillating in xy plane. The external force from the gravitational wave with amplitude $2\sqrt{2}A$ is expressed through the metric tensor h_{jk} as [17]

$$F_j\left(T\right) = \frac{1}{2}\, m\, \ddot{h}_{jk}\, x^k$$

and then reduced to

$$F_x\left(T\right) = \ m\, A\, \omega^2 \sin\omega T \ \ l \sin\varphi \qquad F_y = 0 \tag{1}$$

for polarization states $\varepsilon_+ = \varepsilon_\times = 1/\sqrt{2}$ and sinusoidal profile of the wave. The equation of motion will look like

$$ml\ddot{\varphi} + mg\sin\varphi = F_x\left(T\right) \tag{2}$$

where φ is the angular deviation from the equilibrium point, and l is the length of the pendulum. The suspected values of parameters in (1) are [3]

$$A = 10^{-17} \div 10^{-22} \qquad \omega = 1 \div 10^3\, s^{-1} \tag{3}$$

If we introduce the dimensionless time $t = T/\Omega$, where $\Omega = \sqrt{g/l}$, the equations (2) and (1) lead to

$$\ddot{\varphi} + \sin\varphi = A\nu^2 \sin\varphi \sin\nu t \qquad\qquad \nu = \frac{\omega}{\Omega} \tag{4}$$

The common principal for detecting the GW is based on resonance in the linearized system (4) when $\nu \to 1$. However, we shall consider a dynamic chaos in a nonlinear system. The behavior of system (4) is determined by the

integral of motion, namely the initial energy E_0 [18]. The separatrix value $E_0 = 1$ is the bound between two types of motion: oscillation at $E_0 < 1$ and rotation at $E_0 > 1$. As soon as the external force acts on the system, the energy will not be conserved.

Nevertheless, a weak perturbation caused by a real gravitational wave will not change the character of regular motion (rotation or oscillation) at a relatively small ($E_0 \ll 1$) or very great ($E_0 \gg 1$) initial energy. On the other hand, even a weak perturbation becomes sufficient and may lead to transition from rotation to oscillation (and controversially) for the motion corresponding to small $|E_0 - 1|$. One cannot predict the behavior of the system, whether it oscillates or rotates, for it is determined by the value of energy $E(t_n)$ at the n-th quasiperiod that cannot be known beforehand but must be found step by step for each n. By the way, the quasiperiod grows monotonously with the increase of difference $|E_0 - 1|$, so that a long quasiperiod hints on a favorable condition to set the system into a regime of random motion by a relatively weak perturbation. Thus, one can say that dynamic chaos occurs in the system [18]. The system is inclined to chaos if

$$|E(t_{n-1}) - 1| \le \nu \, |E(t_n) - E(t_{n-1})| \tag{5}$$

where

$$E(t) = E_0 + A\nu^2 J(t) \tag{6}$$

$$J(t) = \int_0^t \dot{\varphi}(\tau) \sin\varphi(\tau) \cos\nu\tau \; d\tau \tag{7}$$

and the n-th quasiperiod is determined by the formula

$$t_n = t_{n-1} + \ln \frac{32}{|E(t_{n-1}) - 1|} \tag{8}$$

while $\varphi(t)$ is given through the elliptic Jacobi function:

$$\dot{\varphi} = \sum_n \frac{2\,(-1)^2}{\cosh(t - t_n)} \tag{9}$$

Contrary to the usual resonance detectors, the limit $\nu \to 1$ is quite insufficient here, since the response of the system on external perturbation, in the right side of (4), occurred as transition to a chaotic regime, takes place in the

sufficiently wide range of external frequencies. Even if ω formidably exceeds Ω or is negligible in respect to the latter, the criterion (5) does not require a great magnitude of the external force. This property is very useful in view of the unknown preset GW frequency being received.

Indeed, a chaos may be set at large n, however, if it is assumed to appear in the first period, we substitute $n = 1$ in Eqs. (5)-(9). The integral (7) can be evaluated analytically in two limit cases, namely,

$$E_1 - E_0 = A\nu Q(\nu) \sin \nu t_1 \tag{10}$$

where $Q(\nu) = 4$ for small ν and

$$Q(\nu) = \nu^2 \sqrt{\pi} e^2 e^{-\pi\nu/2} \tag{11}$$

in the opposite limit. Therefore,

$$|E_0 - 1| \leq \nu A Q(\nu) \sin\left(\nu \ln \frac{32}{|E(t_0) - 1|}\right) \tag{12}$$

For example, at $A = 10^{-20}$, $\nu = 0.1$ and $\nu = 10$ we find from (8), (10)-(12) $t_1 \simeq 50.5$ and $t_1 \simeq 55.8$ respectively. Thus, if the period of the non-perturbed pendulum is approximately $t_1/P \approx 8$ times as large as the period $P = 2\pi$ of the linear oscillator, with the same parameters [$\sin\varphi$ is replaced by a mere φ in Eq. (2)], then the interaction of the gravitational wave with the system will bring dynamic chaos in one cycle. For a sufficiently lesser t_1 (i.e. relatively large $|E(t_0) - 1|$) the chaotic regime does not occur.

Hence, no amplification of the output signal is required, since the system will response to GW by transition to a new regime of motion. Although a high quality is needed: the energy loss during one quasiperiod must not exceed the increment (10). For instance, the thermal noise must not obscure the chaos. In other words the energy fluctuation $\langle \Delta E \rangle$ of the non-linear pendulum must be less than the difference $|E(t_0) - 1|$. A plain calculation by standard thermodynamic formula gives $\langle \Delta E \rangle \simeq 0.5\Theta$, where Θ is temperature. Hence, $\Theta < 2mgl|E(t_0) - 1| \sim mglA \sim AE_0$. This constraint implies $\Theta < 10^{-1}\,^\circ$K for $A = 10^{-20}$, $m = 1$gr, $l = 10\,$cm and terrestrial value of $g = 100\,\mathrm{cm/s^2}$.

So, the interaction of GW with the non-linear pendulum, described by the

equation of motion (2), may be directly observed as transition to a dynamic chaos if the period of non-perturbed pendulum exceeds the definite

value determined by Eqs. (5)-(9). The frequency of GW is not very sufficient and may vary by orders. Of course, the example considered above is not a single variant; the results of the present study can be applied, in principle, to any system where a dynamical chaos is plausible, for instance, even when considering an electron in the crystal lattice [19].

References

[1] J. Weber, Phys. Rev. **117**, 306 (1960).

[2] J. Weber, in *Gravitation and Relativity*, edited by H.Y. Chiu and W.F. Goffmann (Bebjamin, N.Y. 1964), p. 190.

[3] K.S. Thorne, Rev. Mod. Phys. **52**, 285 (1980).

[4] C. Bradaschia *et al.*, Nucl. Instr. Meth. A **289** (1990) 518.

[5] A. Abramovici *et al.*, Science, **256**, 325 (1992).

[6] A. Abramovici *et al.*, Phys. Lett. A **218** (1996) 157.

[7] D. Nicolson *et al.*, Phys. Lett. A **218** (1996) 157.

[8] B. Allen, gr-qc/9607075.

[9] B. Petrovichev, M. Gray, D. McClelland, Gen. Rel. Grav. **30**, 1055 (1998).

[10] V.B. Braginsky and K.S. Thorne, Nature **316**, 610 (1985).

[11] W.W. Johnson and S.M. Merkowitz, Phys. Rev. Lett. **70**, 2367 (1993).

[12] C.Zhou and P.E. Michelson, Phys. Rev. D **51**, 2517 (1995).

[13] J.A. Lobo, Phys. Rev. D **52**, 591 (1995).

[14] G.M. Harry, Th.R. Stevenson, and H.J. Paik, Phys. Rev. D **54**, 2409 (1996).

[15] S.M. Merkowitz and W.W. Johnson, Phys. Rev. D **56**, 7513-7528 (1997).

[16] Th.R. Stevenson, Phys. Rev. D **56**, 564 (1997).

[17] A.D. Speliotopoulos, Phys. Rev. D **51,** 1701 (1995).

[18] G.M. Zaslavski, R.Z. Sagdeev, D.A. Usikov, and A.A. Chernikov, *Week Chaos and Quasi-Regular Patterns*, (Camb. Univ. Press, Cambridge, 1991).

[19] S. Huller and J. Meyer-ter-Vehn, Phys. Rev. A **48**, 3906 (1993).